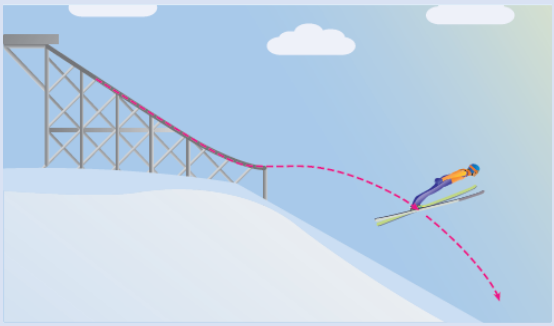
Chapter 6 / **Example 10**

Modelling with polynomials

The path of a ski jumper is illustrated on the diagram on the right.
In order to determine a function to model this jump, the diagram was processed with graphing software to determine a set of points that the path of the ski jumper goes through, x being the horizontal distance from the starting point and y being the vertical displacement from the starting point. The points are listed in the table on the next page.



x	34.8	58.2	83.1	105.9	120.3	141.9	161.4	180	195	206.1	218.4	228.9	239.4
y	–19.5	–34.2	–49.5	–61.8	–64.8	–64.8	–68.1	–74.4	–82.5	–90.6	–101.1	–112.5	–125.7

a Enter the data into your GDC.
b What type of function would model this set of data points and why?
c Use your GDC to determine the model function for this set of data.
d Assess the choice of model by determining the coefficient of determination.
e Plot the model function over the scatter plot and comment on the closeness of fit to the original data.

The straight line that represents the slope of the mountain on which the skier will land goes through the points (0, 0) and (360, -210).

f Determine the equation of the line going through these two points.
g Find the point at which the ski jumper will land on the slope.

From that point on and up to the point where $x = 400$, the skier moves along the straight line describing the slope of the mountain.

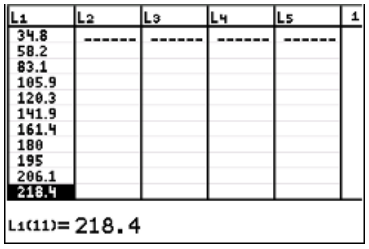
h Write in piecewise form the function describing the whole path of the skier.

Press [STAT] 1:Edit and press [ENTER]

Enter the x-coordinates in the first column.

Press [ENTER] or [▼] after each number to move to the next cell.

Note: *If the list contains other numbers, you can clear it by pressing* [STAT] 4:ClrList *and press* [ENTER]. *The home screen displays* ClrList. *Press* [2nd] [1] [L1] *and press* [ENTER]. *Press* [STAT] 1:Edit *and press* [ENTER] *to return to the table*.



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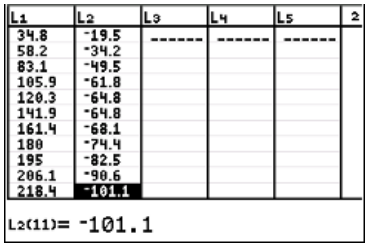
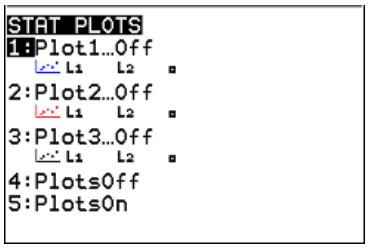
Press ▶ to move to the next column.

Enter the *y*-coordinates in the second column.



Press [2nd] [F1] [STAT PLOT].

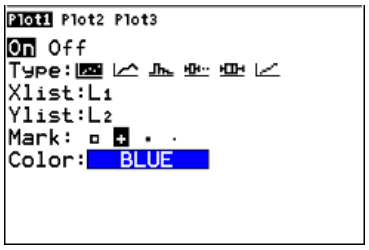
Press [ENTER].



Navigate through the list using ▶ ◀ ▲ ▼ keys.

Select Type ⁘, Xlist L_1 and Ylist L_2 and Mark +. Choose any color.

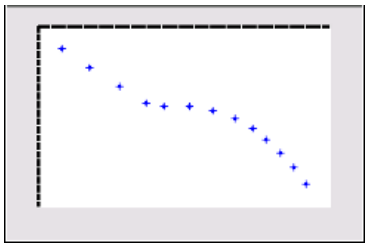
Press [ENTER] after each choice.



Press [F3] [ZOOM] 9:ZoomStat

The GDC displays a scatter diagram of *x* against *y*.

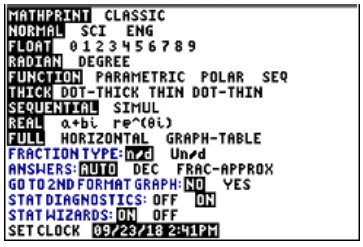
Because the data is approximately cubic, cubic regression is appropriate.



Before calculating the equation of cubic regression, switch the option of calculating the coefficient of determination.

Press [MODE]

Using ▼ and ▼, navigate down to STAT DIAGNOSTICS and select 'ON' by pressing [ENTER].



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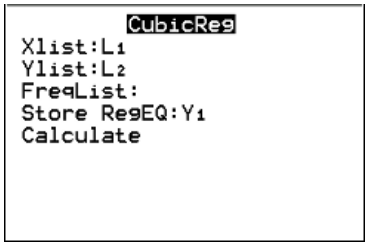
To calculate the equation of cubic regression press [STAT] and [▶] to access the CALC menu.

Select 6:CubicReg and press [ENTER].

Leave the X List as L_1 and the Y List as L_2.

Enter Y_1 in Store RegEQ by pressing [ALPHA] [F4] 1:Y_1

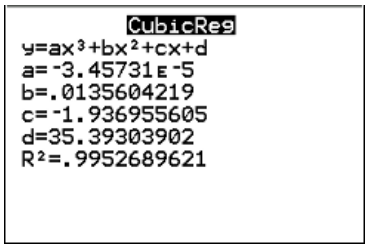
Navigate down to Calculate and press [ENTER].



The cubic curve is given by the equation

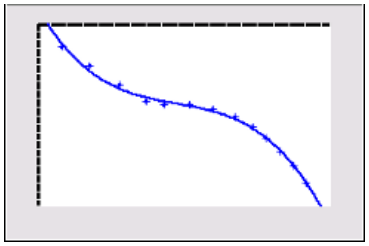
$y = -0.0000346\,x^3 + 0.0136\,x^2 - 1.94\,x + 35.4$

The coefficient of determination is $R^2 = 0.995$, which shows very strong cubic association.



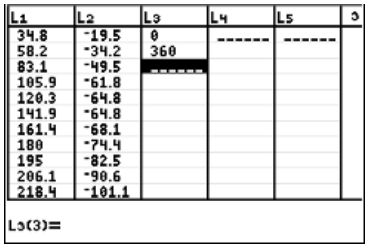
Press [F5] [GRAPH].

The GDC displays the scatter diagram and the regression line.

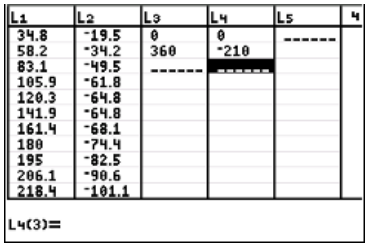


Return to the list editor by pressing [STAT] 1:Edit and press [ENTER].

Move to the next available column and enter the x-coordinates: 0 and 360 in L_3.



Move to the next available column and enter the y-coordinates: 0 and −210 in L_4.



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Modelling with polynomials

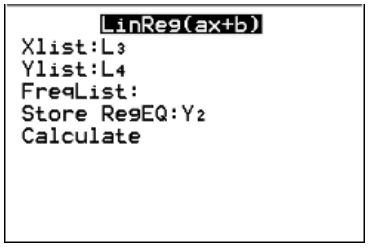
To calculate the equation of linear regression

Press [STAT] and [▶] to access the CALC menu.

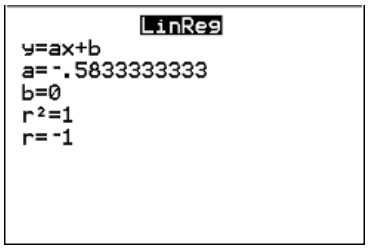
Select 4:LinReg(ax+b) and press [ENTER].

Leave the X List as L_3, the Y List as L_4 and Store RegEQ as Y_2.

Navigate down to Calculate and press [ENTER].



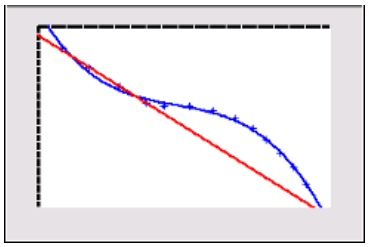
The straight line is given by the equation $y = -0.583x$.



Press [F5] [GRAPH].

The GDC displays the two functions.

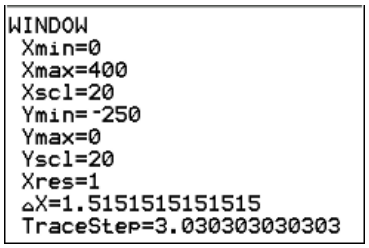
Change the window to show the later part of the skier's path.



Press [F2] [WINDOW]

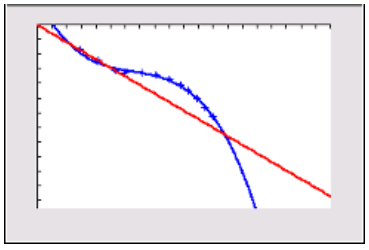
Set the axes to show $0 \leq x \leq 400$ and $-250 \leq y \leq 0$ with scales of 20. Leave the remaining items the same.

Press [F5] [GRAPH] when you have finished.



Press [F5] [GRAPH].

The GDC displays the two functions in a suitable window.

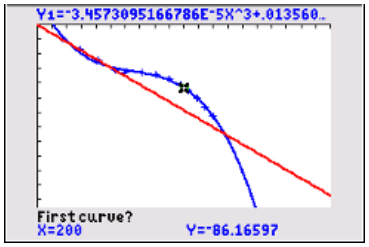


Press [2nd] [F4][CALC] 5:intersect

To find the intersection you need to choose the two lines that intersect.

The GDC shows a cross on one of the lines and 'First curve?'.

Press [ENTER].



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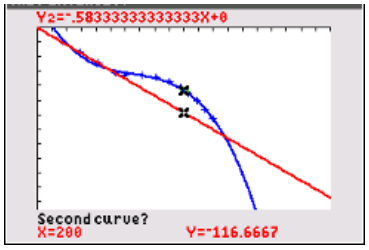
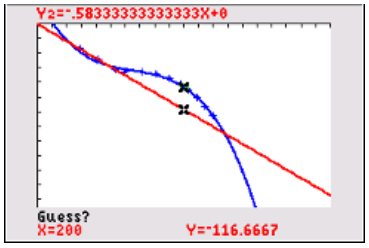
The GDC shows a cross on the other line and 'Second curve?'.

Press ENTER.

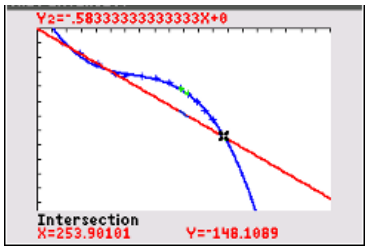


The GDC requires an initial guess for the position of the intersection. Choose the default position.

Press ENTER.



The GDC displays the intersection of the curve and the straight line at the point $(254, -148)$.



The piecewise function is

$$f(x) = \begin{cases} -0.0000346x^3 + 0.0136x^2 - 1.94x + 35.4, & 0 \le x \le 254 \\ -0.583x, & 254 < x \le 400 \end{cases}$$

Press [F1] Y= to display the equation entry screen.

Put both functions in parentheses and modify Y_1 by adding $(0 \le x)(x \le 254)$ and Y_2 by adding $(254 < x)(x \le 400)$

To enter ≤ press 2nd MATH ([TEST]) 6:≤

To enter < press 2nd MATH ([TEST]) 5:<